Comparing Interval Estimates

- Statistical interval estimates are constructed to
  - Estimate parameters
  - Quantify characteristics of population

- To correctly interpret estimates, it must be clearly defined what each interval is estimating
  - Confidence/prediction intervals are well understood
  - Definition of a tolerance interval varies among literature sources

What is a Tolerance Interval?

- Tolerance intervals are being used with more frequency, thus a consistent definition needs to be established

- Definitions found in literature:
  - A bound that covers at least (100-α)% of the measurements with (100-γ)% confidence (Wuipole and Myers)
    - Focuses on where individual observations fall
    - Equivalent to a (100-γ)% CI on middle (100-α)% of Normal distribution
  - An interval that includes a certain percentage of measurements with a known probability (Mendenhall and Sincich)
    - TI is identical to CI, except it attempts to capture a proportion of measurements rather than a population parameter, such as μ

- Computational methods vary depending on author
  - Mee's definition uses non-central t distribution
  - Owen et al. propose to control the percentage in both tails of a distribution
    - No more than a specified proportion lies below or above the TI

- Alternative definition gives interval estimates on lower and upper percentiles, not a percentage, of a distribution
  - lower TI = [X - t(1-α/2,ν)σ/√ν, X + t(1-α/2,ν)σ/√ν]
  - upper TI = [X - t(1-α/2,ν)σ/√ν, X + t(1-α/2,ν)σ/√ν]

Other Interval Estimates

- Simultaneous tolerance interval:
  - Tolerance interval computed for more than one population or sample at a time

- Two 1-sided tolerance interval:
  - 1-sided TI on lower α/2% together with 1-sided TI on upper α/2%, designed to capture (1-α)% of distribution

- Confidence/prediction interval on confidence/prediction interval endpoints:
  - Interval that protects the mean/median confidence/prediction interval endpoints (based on asymptotic Normal distribution of interval endpoints)

- β-content tolerance interval (Mee’s definition):
  - Interval that contains approximately 100β% of the distribution:
    - \( E_{β} = \{ X | P[X < X_{β}] = \beta \} \)
    - \( σ^2 = σ^2 + \hat{σ}^2 = MSB/J/(1-J/MSE) \)
    - \( I = \# \) of batches, runs, blocks, etc.; \( J = \# \) of reps within runs

Evaluation of Interval Estimates

- The relationship among prediction, β-expectation, and β-content intervals is investigated
  - Prediction intervals are compared with β-expectation intervals for models with 1 variance component (all factors fixed)
  - β-expectation intervals are compared with β-content intervals

Prediction vs. β-Expectation Tolerance Intervals

- 1 variance component
  - β-expectation interval equals prediction interval for both overall mean and treatment means (regardless of the number of factors in model)
    - Here \( σ/νt_{α/2} = 0 \) because \( α = 0 \)
    - No df adjustment needed
  - Formula for β-expectation interval reduces to formula for prediction interval

Comparing Interval Estimates

- To understand tolerance intervals and their relationship among other interval estimates for one sample with one variance component, a computer simulation was conducted
  - 50 iterations with data sets of size 50
- Observations were randomly generated, various interval estimates were constructed and compared

Characteristics of Interval Estimates

- Interval estimates can be characterized into 3 groups:
  - Group 1: Bounds on mean or on a CI endpoint
    - lower-upper PICI, lower-upper CICI, lower 1-sided CI, lower 2-sided CI, lower-lower CICI, lower-lower PICI
  - Group 2: Bounds on individual observations, PI endpoints, or upper bound on lower α/2 percentile
    - lower-upper TI, lower-upper PII, lower-upper CIPI, lower 1 sided PI
  - Group 3: Combination of bounds on individual observations, PI endpoints, or lower bound on lower α/2 percentile
    - lower-lower β-expectation (i.e. lower 2-sided PI), 2 1-sided lower TI, lower lower TI, lower SAS TI, lower-lower CIPI, lower 1 sided TI, lower 1 sided TI with Bonferroni correction, lower-lower PII

Conclusions

- Current literature sources offer a wide variety of definitions for tolerance intervals
- Prediction intervals equal β-expectation tolerance intervals for models with 1 variance component
- β-content intervals TI are wider than β-expectation TI
- 1-sided TI is directly related to a 2-sided TI on percentile
- Simulation results indicate
  - Interval estimates can be characterized into 3 groups
  - It must be determined the correct parameter of interest to be captured to determine which estimate to use